# A Softness Distribution Index

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***Abstract*— Although relevant progresses have been done in the last years, modelling soft robots still remains a challenging task, due to the highly nonlinear mechanical behaviour. Different approaches can be chosen for design and control purposes; the choice of the most convenient modelling technique depends on the structural properties of the given soft robot (i.e., on both its material and its geometry). In this paper, we introduce a Softness Distribution Index (SDI) as a useful tool for the soft robotics researcher to select an appropriate modelling technique. To present the concept in a clear and concise way, here we focus on the planar bending of a single soft link, showing that the SDI provides information about the distribution of the softness. Future works will address the definition of a comprehensive set of indices, to take into account all possible deformation modes in a three-dimensional space.**

## I. INTRODUCTION

Modelling the mechanical response to forces (gravity, applied loads, inertia, pressure, etc.) is crucial for both rigid-linked and soft robots. Conventional rigid-linked robots are used for industrial applications such as manipulation tasks, pick-and-place operations, handling of heavy metal parts, and many others; they are designed to be notably *stiff*. They are typically built with high Young's modulus material (e.g., steel, aluminium, metal alloys) and are not intended to undergo large strains.

On the contrary, soft robots are designed to be highly deformable. Their highly nonlinear behaviour introduces relevant difficulties in the modelling. Finite elements simulations, Cosserat models, simplified equations including few parameters have been proposed in literature ([1], [2], [3], [4], [5]) to tackle the problem. Researchers agree that the field of soft robotics still lacks a unified modelling approach [6],[7]. This is due to the fact that soft robots can have a wide variety of structures, actuations, materials. The mechanical behaviour is always nonlinear, but significant differences can be found between two soft robots. For instance, both the Octopus arm [3] and the gripper [8] are considered soft, but their mechanical behaviour cannot be described by the same modelling technique in a convenient way. For any soft-bodied robot with certain material, geometry, and operational conditions, different issues can arise in modelling, due to variable contact conditions, uncertainty about the material properties [9], or the occurrence of large strains. Among the mentioned issues, the large strains play a key role in modelling soft bodied robots. The computation of large strains must be carefully addressed when using finite elements, since they may result in excessively distorted mesh, leading to a failed simulation. Moreover, not all the formulations that can be adopted to run nonlinear FE analysis are suitable to compute large strains (see, e.g., [10]). More than for rigid-linked robots, appropriate choices in the modelling for a soft robot are crucial to perform the computation with desired accuracy and affordable computational burden. To guide roboticists in these choices, we suggest that it is useful to consider not only *how much soft* is the robot, but also *how the softness is distributed* throughout the whole structure.

Recently, some authors provided a novel definition of softness ([11]), which should be useful to design a soft robot for a specific application. Following their work, in this paper we propose a mathematical definition of softness, based on a novel index to measure the distribution of softness along the body of the robot. This index helps the researcher in the choice of the appropriate modelling technique. In particular, in Section II we shortly discuss the factors that determine the softness of a structure. In [12], soft robots are categorized into four types of deformation behaviour: longitudinal deformation, bending, flowing and transformation. Adopting the same viewpoint, we propose the use of an index for each of these classes that takes into account the distribution of the softness in the structure; in Section III, we define the index for the case of planar bending. Conclusions follow.

## II. STIFFNESS OR SOFTNESS?

Stiffness is a property of structures and it is related to both the material and the geometry. In this paper, we use the term *softness* as complementary to *stiffness*. Actually, stiffness is a physical quantity; by the term *soft* we refer to a body, or robot, or structure, having *low stiffness*. Stiffness is defined in relation to an applied load and the consequent deflection; it is indeed a relative quantity. For what we said, the same consideration applies to softness.

For the purpose of this paper, let us consider a straight cantilever beam and let be $x$ the distance between the clamped end section of the beam and the generic cross section (Fig. 1). If the Young's modulus $E$ and the moment of inertia of the cross-section $J$ are variable along the longitudinal axis, they can be written as functions of $x$:

$$E = E\,(x) \tag{1}$$

$$J = J\,(x) \tag{2}$$

From the classical beam theory, the product $EJ$ is known as the flexural rigidity of the beam. By virtue of (1) and (2),

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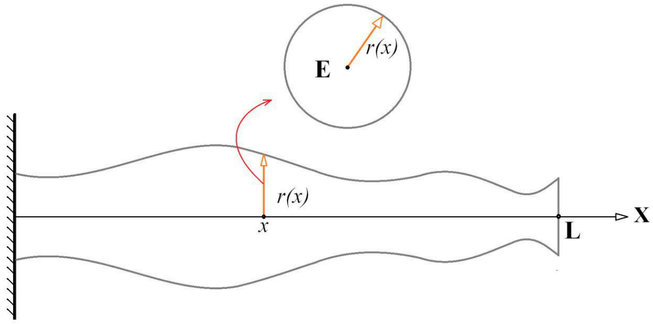


Fig. 1. Cantilever beam with variable cross section of radius $r(x)$ and variable Young's modulus $E(x)$

this quantity depends on $x$:

$$k_f(x) = E(x)\,J(x) \tag{3}$$

For a beam subject to a moment $M(x)$, bending with curvature $\rho(x)$, the quantity $k_f$ appears in the relation

$$\rho(x) = \frac{M(x)}{E(x)J(x)} \tag{4}$$

## III. AN INDEX FOR FLEXURAL SOFTNESS DISTRIBUTION

Let both $E$ and $J$ be smooth functions in the interval $[0, L]$, where $L$ is the length of the beam. The first derivative of the flexural rigidity $k_f$ is given by

$$\alpha(x) = \frac{dk_f(x)}{dx} = J(x)\frac{dE(x)}{dx} + E(x)\frac{dJ(x)}{dx} \tag{5}$$

Let be $x_M$ the coordinate at which $\alpha(x)$ is maximum, and $x_m$ the one at which it is minimum. We define the Softness Distribution Index (SDI) as

$$\text{SDI} = \frac{\alpha(x_M) - \alpha(x_m)}{max\{|\alpha(x_M)|, |\alpha(x_m)|\}} \cdot \frac{L}{x_M - x_m} \tag{6}$$

For a beam made of a single material and with constant cross section, the function in (5) is null everywhere, and the SDI cannot be computed since the denominator in (6) would be equal to zero. The same holds if both $E(x)$ and $J(x)$ are variable but their product is constant.

Let us now consider two cantilever beams, B1 and B2, both having Young's modulus $E$ and circular cross section with variable radius $r_1(x)$ and $r_2(x)$ respectively, such that they are characterized by different functions $J_1(x)$ and $J_2(x)$. Here we choose the two profiles shown in Fig. 2, having respectively:

$$r_1(x) = r_M + \frac{1}{2}(r_M - r_m)\left(\cos\frac{n\pi(x - \frac{L}{2} - \frac{L}{n})}{L} - 1\right)\cdot \tag{7}$$

$$\cdot\, H\left(x - \frac{L}{2} - \frac{L}{n}\right) H\left(\frac{L}{2} + \frac{L}{n} - x\right)$$

$$r_2(x) = r_M e^{-\frac{x}{L}} \tag{8}$$

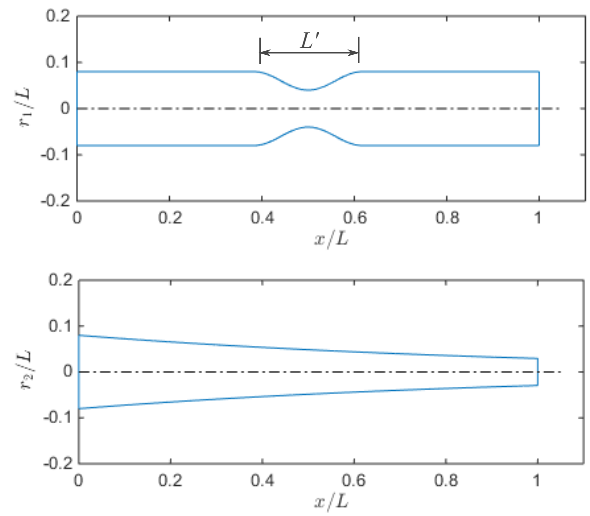


Fig. 2. Side view of beams B1 (top) for $n = 8.5$ and B2 (bottom).

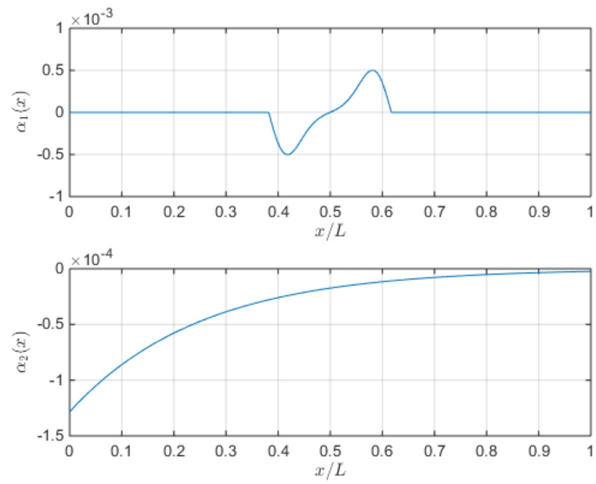


Fig. 3. Functions $\alpha_1(x)$ (top) and $\alpha_2(x)$ (bottom) for beams B1 and B2 represented in Fig. 2.

where $r_M$ and $r_m$ denote the maximum and minimum radius respectively and $H(\cdot)$ is the Heaviside function. All the parameters are here normalized with respect to $L$. The geometrical parameter $n$ determines the length of the bottleneck, which is $L' = 2L/n$. If $n$ increases, the length $L'$ decreases and the groove in B1 becomes narrower; for B1 represented in Fig. 2 it is $n = 8.5$. $E$ is chosen unitary, since in this case it is merely a multiplying factor. The first term in (5) vanishes and the function reduces to

$$\alpha_i(x) = \frac{E\pi}{4}\frac{d\left(r_i^4(x)\right)}{dx} \tag{9}$$

with $i = 1, 2$.

Fig. 3 shows the functions $\alpha_1(x)$ and $\alpha_2(x)$. The trend of $\alpha_1(x)$ highlights an abrupt variation of $k_{f_1}$, as intuitively expected. The trend of $\alpha_2(x)$, instead, shows that $k_{f_2}$ decreases from the clamped end to the free one (being $\alpha_2(x)$

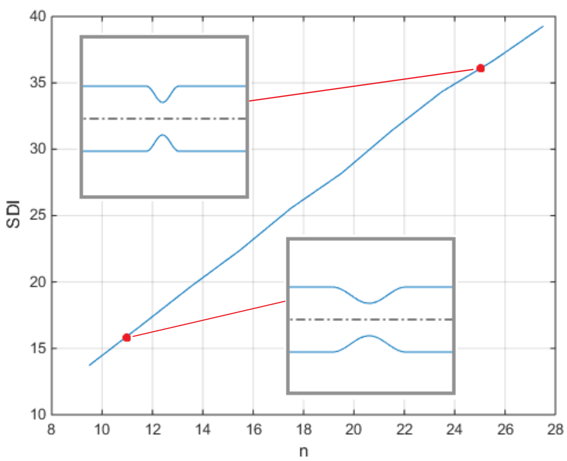


Fig. 4. SDI for beam B1 of Fig. 2 vs. $n$. Two different geometries are shown in the insets: for $n = 11$, it is SDI $= 15.9$; for $n = 25$, it is SDI $= 36.6$.

negative) without any spike.

It is computed that SDI for B1 is equal to 12.21, being $x_{M_1} = 0.518$, $x_{m_1} = 0.418$, $\alpha_1(x_{M_1}) = 5.017 \cdot 10^{-4}$ Nm, $\alpha_1(x_{m_1}) = -5.017 \cdot 10^{-4}$ Nm. The SDI for B2 results equal to 0.98, being $x_{M_2} = L$, $x_{m_2} = 0$, $\alpha_2(x_{M_2}) = -2.36 \cdot 10^{-6}$ Nm, $\alpha_2(x_{m_2}) = -1.284 \cdot 10^{-4}$ Nm.

The SDI provides information about abrupt variation of $k_f$ along the structure. Under a moment $M$ applied to the free end section, both beams deflect, but with different distribution of the curvature along the longitudinal axis: the largest contribution to the deflection of the free end of beam B1 is the large rotation occurring at the bottleneck, where $r_1$ is minimum; instead, beam B2 deflects with monotonically increasing curvature. We observe that a SDI $\gg 1$ detects the presence of abrupt variations of the quantity $k_f$, i.e., there is a cross section of the beam which behaves like a hinge. In this case, a Pseudo Rigid Body Model (PRBM) (see, e.g., [13]) might be accurate enough to describe the mechanical behaviour of the beam for a large range of applied load. For structures having instead SDI $\simeq 1$, as B2, Cosserat models seem more appropriate, or finite elements can be used if large strains occur due to large deflection of the free end.

Further observations concerning the value of SDI can be done by varying the value of $n$ in the expression for $r_1(x)$ and keeping unchanged all the other parameters ($L$, $r_M$ and $r_m$). The greater $n$, the more abrupt appears the variation of $J_1(x)$, since the bottleneck occurs in a shorter length $L'$; this is related to an increase of the SDI, as shown in Fig. 4.

It is important to notice that the value of SDI as defined so far is not sufficient to assess whether the body can be considered *soft* or not, according to the current approach in the field of soft robotics; for this purpose, the order of magnitude of $k_f$ should be taken into account as well. For instance, beam B2 might be made of steel and have a large cross section, such that it would not be useful for soft robotic applications. SDI describes the distribution of the softness (or, stiffness) along the structure, but does not provide information on the order of magnitude of this quantity. Therefore, the use of SDI as defined in (6) is meaningful provided that the structure under investigation is considered *soft*. In future works we will tackle this aspect by defining an index which depends on the order of magnitude of $k_f$.

## IV. CONCLUSIONS

In this work we have introduced a novel index for measuring the distribution of the softness along the soft bodied structures. SDI takes into account simultaneously both effects of geometry (abrupt change in thickness of the body) and of the material (Young's modulus) of the structure.

We suggest that a set of indices describing the distribution of the softness in a structure would represent a useful tool for the design of soft robots and for the choice of a convenient modelling technique, also for control purposes. In this paper, SDI for planar bending has been defined, in order to introduce the concept and provide some examples.

To summarise, the SDI is a real number. Dividing the axis of real numbers $\mathbb{R}$ in $m$ intervals $\{\mathbb{I}_1, \mathbb{I}_2, \cdots, \mathbb{I}_m\}$ such that $\bigcup_{i=1}^{m} \mathbb{I}_i = \mathbb{R}$, the goal is to associate one modelling technique to each of these intervals. In this way, the SDI would highlight a relation between the structure and the modelling technique and would allow the roboticists to choose the most convenient mathematical description. A deeper study will be performed to relate each modelling technique to an interval of real values of SDI. Future works will attempt to define a comprehensive set of indices, that should account for all the deformation modes other than the bending, as well as for the loading conditions and the order of magnitude of the stiffness of the structure.

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