

# Efficient Reduced-Order Models for Soft Actuators

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**Abstract**—Soft robotics have gained increased attention from the robotic community due to their unique features such as compliance and human safety. Impressive amount of soft robotic prototypes have shown their superior performance over their rigid counter parts in healthcare, rehabilitation, and search and rescue applications. However, soft robots are yet to capitalize on their potential outside laboratories and this could be attributed to lack of advanced sensing capabilities and real-time dynamic models. In this pilot study, we explore the use of high-accuracy, high-bandwidth deformation sensing via fiber optic strain sensing (FOSS) in soft bending actuators (SBA). Based on the high density sensor feedback, we introduce a reduced order kinematic model. Together with cubic spline interpolation, this model is able to reconstruct the continuous deformation of SBAs. The kinematic model is extended to derive an efficient real-time equation of motion and validated against the experimental data.

## I. INTRODUCTION

The rise of bioinspired soft robotics demands highly compliant and inherently safe actuators instead of precise and fast ones that are desirable in traditional rigid-bodied robotics. In this paper, we focus on soft bending actuators (SBA) that bends actively or passively, during operation [1]. Lately, soft robots, in the sense that “completely” deforms, have been an active area of research. Soft robots have often been constructed from soft materials such as elastomeric polymers. This enables continuous deformation with a few active degrees of freedom (DoF) to form complex shapes (compare to fixed “geometric shapes” of rigid-bodied robots). Due to the passive deformation these robots undergo, they are considered as infinite DoF systems. But in reality, they only have a handful of actuated DoFs, and therefore are highly underactuated. Many soft robot prototypes have been proposed over the years which employed actuation methods such as pneumatic, hydraulics, shape memory alloys, electroactive polymers, and magnetic fluids. By systematically organizing SBA, these prototypes demonstrated manipulation, snake locomotion [2], legged locomotion [3], and peristaltic locomotion [4].

Despite the increased amount of research in SBA, they are yet to make an impact outside laboratory settings. The authors believe that this is due in part to the lack of adequate models that can be used in real-time and advanced sensing techniques (suitable for soft robots) to provide accurate shape deformation

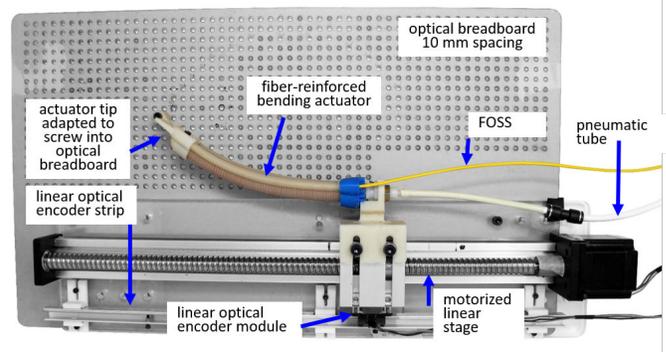


Fig. 1. The prototype soft bending actuator (SBA) experimental setup with the integrated fiber optic strain sensor (FOSS).

data in real-time. Much of the modeling approaches that have been presented to-date involves systematic derivation of system quasi-statics based on the mechanical properties of SBAs [1], [5]. These models define the deformation as a function of pressure and thus only captures the steady state information. Yet, pneumatic actuators are highly compliant and therefore can deform into complex shapes as a result of momentum of fast actuation or external forces. Thence, such static models are of limited use in applications where speed dominates the system behavior and raises the need for better and dynamic models.

On top of that, the sensors utilized in soft robots so far have not been able to reap the benefits of soft robots. For instance, image based sensing, capable of measuring the entire shape deformation in real-time, requires specific backgrounds and line-of-sight to be reliable in automated image processing and thus limits the robot’s application space. Other sensing methods include highly nonlinear bending sensors and strain sensors that are unreliable in practice. When obtaining the state feedback, the most common approach is to measure the tip coordinates. But, it is not a good indicator of the soft robot as it can be in any one of state (due to compliance) out of infinitely many possibilities. In this respect, fiber optic strain sensors (FOSS) could serve as high-fidelity position sensors for soft robots.

In this pilot study, a high resolution FOSS sensor array is integrated into a fiber-reinforced SBA to obtain accurate position at 0.8 mm along the SBA at 75 Hz. This data density is beyond what soft roboticists have been able to collect from soft robots so far and thus opens up new research avenues on how this shape data may be used to enable applications previously deemed infeasible for soft robots. In addition to introducing the FOSS for soft robotics, we also introduce a

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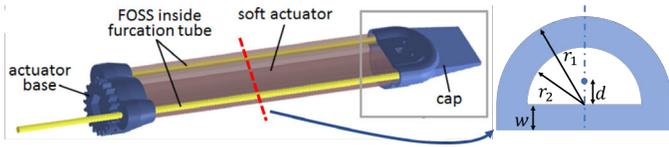


Fig. 2. (Left) Integration of fiber optic strain sensor into the soft bending actuator, and (Right) the cross-section of the SBA.

reduced-order dynamic model, that matches the FOSS data, for SBA as a compromise among numerical efficiency, accuracy, and complexity.

## II. PROTOTYPE SOFT BENDING ACTUATOR WITH INTEGRATED SENSING

This pilot study was conducted in the Luna Innovations Inc, headquarters in Blacksburg, VA. The FOSS sensor we used in our experiment is based on Optical Frequency Domain Reflectometry (OFDR) [6]. The readers are referred to [7] for an in-depth discussion of the FOSS sensing technology. Several features worth noting about the FOSS platform are: (a) fiber optic cable is compliant, (b) bendable in curvature as low as 10 mm, (c) can function under significant morphological variation, (d) can detect bending and twisting along the entire length, (e) produce full (translation and rotation) spatial data at every 0.8 mm, (f) sampling rates up to 250 Hz, and (g) eliminates the need for line-of-sight and thus, can be integrated into soft material robotic structures.

The hardware experimental setup used in this pilot study is shown in in Fig. 1 which is controlled via a Matlab Simulink Realtime System. A digital proportional valve was used to supply pressure the actuator at various pressures. A pressure sensor was used to record the pressure readings. The details of the fabrication process of the SBA used here (see Fig. (2)) can be found in [8]. SBA bending portion is 17 cm long, weights 69 g, and has semi-annular cross section. The fiber-reinforced wall thickness,  $w$ , is estimated to be 4 mm with  $r_1 = 14 mm$  and  $r_2 = 10 mm$ . The reinforcement fibers around the bladder increase the operating pressure range and thus the resulting output force. To maintain the FOSS without significantly straining and affecting the natural compliance of SBA, we co-molded a low-friction Teflon lined lumens around the edge of the SBA body (see Fig. 2). The FOSS was housed within a 3 mm diameter furcation tube (Fiber Instrument Sales, Inc., Oriskany, NY, part #: F00FR3NUY) and restrains the sensor from bending beyond 10 mm radius of curvature. As FOSS has no markers, it is a challenge to determine the end points of the SBA. To solve this problem, we attached a cap with a constant curvature at the tip of the SBA. By identifying the curvature and its length, we could then find the tip location from the experimental data. We then rigidly attached the SBA with the FOSS sensor to a fixed table in the FOSS's XY plane and gather dynamic data.

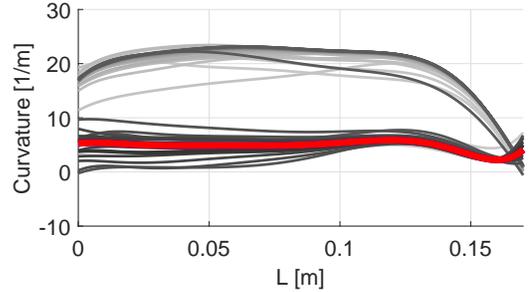


Fig. 3. Curvature profile of SBA during a 240 kPa step input.

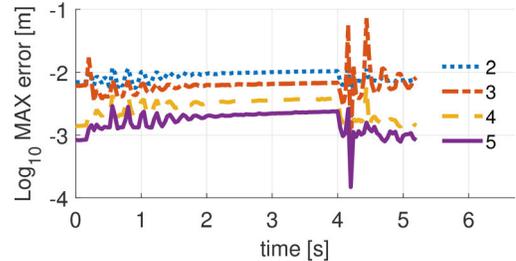


Fig. 4. Maximum error comparison for 2-5 point approximation. The 5-link system resulted 0.0024 m maximum error and 0.0012 m mean error for the duration of the experiment.

## III. METHODOLOGY

### A. Reduced-Order Model

To-date, SBA models consider uniform material distribution and therefore simulate bending as circular arcs (constant curvature). However, SBA fabrication processes are prone to variations in physical and mechanical parameters (material density and thickness) and results in non-uniform bending. Figure 3 shows the curvature variation (initial value in red) along the SBA during motion. Hence, a new model that could account for such deviations need to be proposed.

Ideally, a high-DoF, discrete jointed system would be capable of capturing the nonlinear deformation. But, analogous to finite element models, they are numerically inefficient for real-time applications. Here, we combine a low-order discrete system and cubic spline interpolation to represent the smooth and continuous bending. We utilized experimental data, collected from the SBA for a step response, to determine the lowest order discrete model capable of acceptable shape reconstruction as follows. The FOSS data at any time contains Cartesian (XYZ) trajectories for 212 points (0.8 mm apart) along the entire length. We divided the length of SBA to 2-5 similar-length segments (from each time instance) and applied cubic spline interpolation to reconstruct the deformation shape. We used the maximum Euclidean distance between the reconstructed curve and the actual shape as the measure of similarity between the curves. Figure 4 shows the error progression for discrete jointed systems of 2-5 rigid segments. The 5-link system reconstructed the bending shape with less than 3 mm maximum error (and 1.3 mm mean error) whereas low order discretizations lead to large errors (spikes around 4.1 s).

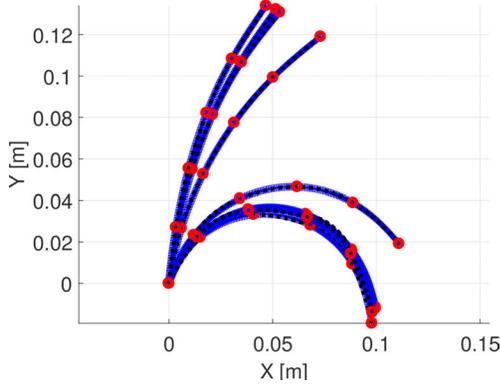


Fig. 5. The difference between the measured and the reconstructed deformation shapes of the SBA deformation during bending (Fig. 4). + experimental data, o joints of the reduced-order model and the solid line shows the cubic spline interpolation.

Figure 5 shows some instances of this experiment where we compare the actual data (+ marks) and the reconstructed curve (solid line) along with the discrete points we used to reconstruct the shape. It could be seen that the shape reconstruction is identical without any noticeable departure from the measured curve. Thus, we will use a 5-link discrete approximation for the development of the dynamic model.

### B. Kinematic Model

The schematic of a planar 5-link discrete system model is shown in Fig. 6-A. Let us consider any  $i^{th}$  link of the discrete model (Fig. 6-B). Note that, since we define the discrete joints based on the FOSS data, the link lengths ( $l_i$ ) are not identical. As shown in Fig. 1 the unactuated SBA has a noticeable curvature (see Fig. 1). Thus, each link has an angle offset  $\theta_{i0}$  (also not uniform) relative to the previous segment while  $\theta_i$  is the joint-space variable. The proposed approach attempts to find a middle ground (with 5 DoF) to emulate an infinite DoF SBA system with minimal error while facilitating real-time dynamics.

The standard homogeneous transformation techniques can be applied to derive the homogeneous transformation matrix (HTM) of any  $i^{th}$  segment,  $\mathbf{T}_i \in \mathbb{SE}^2$  is given by (1) where  $\mathbf{R}_z \in \mathbb{SO}^2$  is the rotational HTM about the +Z axis and  $\mathbf{P}_y \in \mathbb{R}$  is the translational HTM along the +Y axis.  $\mathbf{R}_i \in \mathbb{R}^{2 \times 2}$  and  $\mathbf{p}_i \in \mathbb{R}^{2 \times 1}$  are the rotational and translation matrices of the segment with respect to the local coordinate system,  $\{O_i\}$ .

$$\mathbf{T}_i(\theta_i) = \mathbf{R}_y(\theta_i + \theta_{i0}) \mathbf{P}_z(l_i) = \begin{bmatrix} \mathbf{R}_i & \mathbf{p}_i \\ 0 & 1 \end{bmatrix} \quad (1)$$

Employing the standard serial link kinematics, the HTM of the  $i^{th}$  link,  $\mathbf{T}^i : \mathbf{q}_i \mapsto \mathbb{SE}^2$ , with respect to the task-space,  $\{O\}$ , is given by (2) where  $\mathbf{q}_i = [\theta_1 \dots \theta_i]^T \in \mathbb{R}^i$  is the joint-space vector and  $\mathbf{R}^i \in \mathbb{R}^{2 \times 2}$  and  $\mathbf{p}^i \in \mathbb{R}^{2 \times 1}$  are the segment rotational and translation matrices relative to  $\{O\}$ .

$$\mathbf{T}^i = \prod_{k=1}^i \mathbf{T}_k = \begin{bmatrix} \mathbf{R}^i & \mathbf{p}^i \\ 0 & 1 \end{bmatrix} \quad (2)$$

Expanding (2), the recursive relationships for  $\mathbf{R}^i$  and  $\mathbf{p}^i$  can be derived as

$$\begin{aligned} \mathbf{R}^i &= \mathbf{R}^{i-1} \mathbf{R}_i \\ \mathbf{p}^i &= \mathbf{p}^{i-1} + \mathbf{R}^{i-1} \mathbf{p}_i \end{aligned} \quad (3)$$

where  $\mathbf{R}^{i-1}$  and  $\mathbf{p}^{i-1}$ , according to our definition, is the tip coordinates of the prior,  $(i-1)^{th}$  segment.

We take the time derivative of (3) to calculate the body velocity (i.e., with respect to  $\{O_i\}$ ) of the  $i^{th}$  segment in recursive form as

$$\begin{aligned} \boldsymbol{\Omega}_i &= \mathbf{R}_i^T (\boldsymbol{\Omega}_{i-1} \mathbf{R}_i + \dot{\mathbf{R}}_i) \\ \mathbf{v}_i &= \mathbf{R}_i^T (\mathbf{v}_{i-1} + \boldsymbol{\Omega}_{i-1} \mathbf{p}_i + \dot{\mathbf{p}}_i) \end{aligned} \quad (4)$$

where  $\boldsymbol{\Omega}_i \in \mathbb{R}^{2 \times 2}$  and  $\mathbf{v}_i \in \mathbb{R}^{2 \times 1}$  are the skew symmetric angular velocity matrix and linear velocity vector respectively [9]. The angular velocity,  $\boldsymbol{\omega} \in \mathbb{R}$  can be derived from  $\boldsymbol{\Omega}_i$ , as  $\boldsymbol{\omega}_i = [\boldsymbol{\Omega}_i]_{12}$ .

### C. Dynamic Model

The kinematic relationships derived in III-B are used to derive the equations of motion (EoM) in standard form given by (5). A detailed treatment of the recursive numerically efficient dynamic formulations, the readers are referred to [10].

$$\mathbf{M} \ddot{\mathbf{q}} + (\mathbf{C} + \mathbf{D}) \dot{\mathbf{q}} + \mathbf{K}_b \mathbf{q} = \boldsymbol{\tau} \quad (5)$$

where  $\mathbf{M}$ ,  $\mathbf{C}$ ,  $\mathbf{D}$ ,  $\mathbf{K}_b$ , and  $\boldsymbol{\tau}$  are the generalized inertia matrix, centrifugal/Coriolis force matrix, damping force matrix, torsional spring coefficient at the joints, and the joint-space torque applied to the system. From the semi-annular cross-section of SBA bladder, we can compute the external

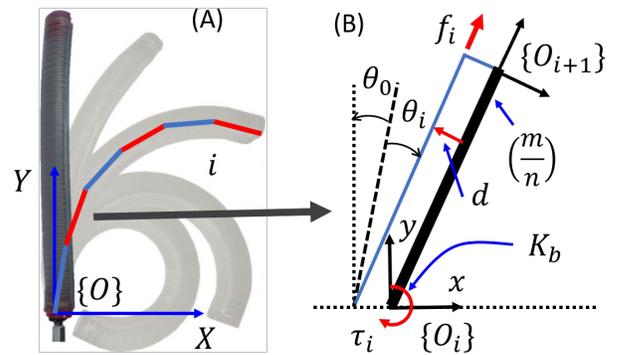


Fig. 6. (Left) the discrete jointed model of SBA in the task-space coordinate system,  $\{O\}$ , (Right) the segment parameters in the local coordinate frame,  $\{O_i\}$ , and related physical properties (further detailed in Section III-C).

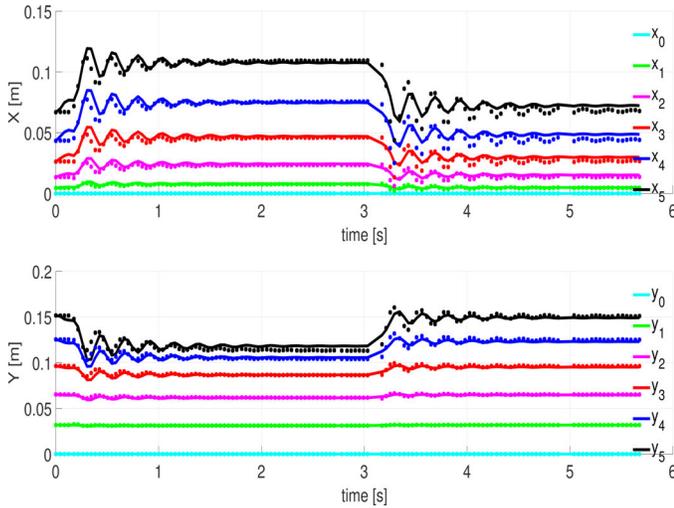


Fig. 7. Comparison of the numerical results with the experimental data. The subscripts of the X,Y coordinates denotes the task-space trajectories of the tips of segments in the task-space,  $\{O\}$ .

torque,  $\tau_i = pAd$  where  $d = \frac{4r_2}{3\pi}$  ( $d$  shown in Fig. 6-B),  $p$  is pressure, and  $A = \frac{\pi r_2^2}{2}$  is the SBA cross-section area.  $K_b$  and  $\mathbf{D}$  are difficult to estimate and therefore found via experimental characterization process, similar to approach reported in [10] to obtain the optimal  $K_b = 1.6067$  and  $\mathbf{D} = 10^{-3} \text{diag}([8, 8, 8, 8, 8])$ . The EoM is then implemented in Matlab Simulink platform and solved using the ODE15s solver.

#### IV. EXPERIMENTAL RESULTS

Figure 7 shows the step response comparison of EoM given in III-C and experimental data for a rectangular pressure input of 119 kPa for 2.76 s duration starting at 0.12 s. The plot shows the X and Y coordinate trajectories of the 5 tip positions of the reduced order discrete link dynamic model (numbered from 1-5). Empirical results demonstrates that the numerical model simulates the SBA well overall. The abrupt pressure increase causes the SBA to move suddenly causing the entire arm to bend and oscillate (due to momentum and high compliance). The system is under-damped with oscillation frequency about 5 Hz and achieve steady state due to the friction of the system. It is worth nothing that, the SBA models proposed so far yet to model dynamic behavior at such high frequencies. The proposed model successfully simulates the SBA response well overall, specifically, the overshoot and vibration frequency. In addition, the model is able to well model the under-damped oscillations, not only at the tip, but also along the length of the arm. We can observe some steady stat errors towards the end of the simulation. These can be attributed to the inherent hysteresis of SBAs. The ongoing work explores the feasibility of incorporating the hysteretic behavior of SBA using the model reported in [11].

#### V. CONCLUSIONS AND FUTURE WORK

Soft bending actuators have shown strong potential powering the next generation soft robotics for applications in non-engineered and human-friendly spaces. So far, that potential has not been realized. This is due to the lack of sophisticated feedback systems and advanced dynamic models. In this paper, we integrate a SBA with FOSS sensor to obtain high fidelity data of the entire SBA up to 250 Hz rate and demonstrate orders of magnitude better look at the SBA state-space than the prevailing sensing techniques. To meet the demands of accuracy and numerical efficiency, we systematically derived and proposed a reduced-order dynamic model. The simulated output was then examined against the FOSS data to validate the model. The proposed model exhibited good agreement in capturing the dynamics of SBA. Future work will focus on implementing hysteresis and dynamic control of SBAs for applications in environmental sensing [12] and contact detection.

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